

## SUPPLEMENT TO "NATURE."

## THE ENTROPY OF RADIATION.

*Vorlesungen über die Theorie der Wärmestrahlung.*  
By Dr. Max Planck. Pp. viii+222. (Leipzig:  
Johann Ambrosius Barth, 1906.) Price 7 marks.

A QUANTITY of heat  $Q$  is transferred by radiation from a body the surface of which is at temperature  $T_1$  to a body the surface of which is at a lower temperature  $T_2$ . From this cause alone the former body loses a quantity of entropy  $Q/T_1$ , while the latter gains a quantity of entropy  $Q/T_2$ . The net gain of entropy of the bodies arising from the transaction is  $Q(1/T_2 - 1/T_1)$ . Where and how does this gain take place?

Some people have expressed the view that the notion of entropy has no place in radiation phenomena, and that it is only a convenient symbol introduced for the purpose of representing a certain class of heat phenomena occurring in material bodies. But the entropy gained or lost by a body measures the gain or loss of unavailable energy on the supposition that energy can be converted into work by means of ideal reversible engines working between the body and an indefinite medium at unit absolute temperature. It will thus be seen that the change of entropy above considered represents a definite amount of what Mr. Swinburne calls "incurred waste," a change which cannot be undone, which leaves an indelible imprint on the state of the universe, which represents a loss of availability, or, from an engineering point of view, a loss of value. We might say that, though the energy  $Q$  has not been altered in amount, it has become a less marketable commodity by the change. It thus becomes important to examine exactly where and how the change of entropy has taken place, that is, to extend the notion of entropy to the ether.

If we begin by attempting to apply reversible thermodynamics to the ether, we arrive at a single result only, namely, Boltzmann's differential equation connecting Maxwell's formula for the radiation pressure with Stefan's law for the intensity of radiation inside a black cavity. For this particular kind of radiation entropy is fully defined, and the energy per unit volume being proportional to the fourth power of the absolute temperature, the entropy is proportional to the cube, being  $4/3$  of the energy divided by the temperature.

In this case there is no violation of the relation between unavailable energy and entropy which forms the basis of the thermodynamics of a material body. At the temperature of the cavity the unavailable energy represented by the entropy is  $4/3$  of the total energy, but the discrepancy is accounted for by the work of expansion against the radiation pressure. Further, as the author shows, the gain of entropy when communication is established between two black cavities at unequal temperatures is calculable by ordinary thermodynamic methods, just as is the gain of entropy produced by diffusion of two portions of gas at unequal pressure or temperature or both.

Irreversible changes will necessarily occur at the surface of a body unless either the surface is perfectly reflecting or the incident radiation in the ether is of the character of black-cavity radiation; for the radiation emitted by a body is necessarily distributed in all directions, while it can absorb radiation falling on it in particular directions.

If, on the other hand, a uniformly heated black body is radiating heat into space, the radiation received at an external point will be limited in direction by the solid angle which the body subtends at that point, and this will decrease as the distance from the body increases, but no passage of heat from a hotter to a colder body is necessarily associated with the outward propagation of the radiation. We may imagine an ideal perfectly reversible burning glass capable of concentrating the radiation on a receiving body in such a way that it converges from all directions on the body, the solid angle formed by the directions being thus increased to  $2\pi$ . If the radiating and receiving bodies are perfectly black, the latter will be in a state of thermal equilibrium with the ether if its temperature is equal to that of the radiating body, and the radiation may thus be absorbed at the temperature of emission by perfectly reversible methods.

This does not mean that the outward propagation of radiation from a finite body is reversible, for if a body, say a sphere, commences to radiate into infinite space previously devoid of radiation, available energy is lost in consequence of the radiation pressure set up. If, now, we imagine the sphere surrounded by a concentric perfectly reflecting sphere, and suppose that at the surface of this latter the energy of radiation per unit volume is  $\psi$  and radiation pressure  $f$ , then, if the volume of the sphere is decreased by  $dV$ , the sphere will have to absorb heat-energy  $\psi dV$  which is unavailable at the temperature  $T$  of the sphere, and, moreover, available energy  $f dV$  will have to be supplied in order to overcome the radiation pressure. Hence it appears that even in this case the entropy per unit volume at any point of the ether assumes the form  $(\psi + f)/T$ , where  $T$  is the black-body temperature corresponding to the same intensity of radiation per unit solid angle. And as the radiation proceeds outwards the quantity  $f dV/T$  represents the gain of entropy over and above the quantity of entropy taken from the radiating sphere which is given by the  $dQ/T$  formula.

These introductory statements will give some idea of the difficult task which Dr. Planck has undertaken in his endeavour to trace the connection between radiation phenomena and the assumed principles of irreversible thermodynamics. So many physicists have given up this task as hopeless that Dr. Planck has had to rely, to a large extent, on his own investigations; and the list of original papers, published between 1896 and 1902, affords an insight into the amount of time and thought the author has given to the subject in its many and varied aspects. The present book, based as it is on the courses of lectures delivered by Dr. Planck at Berlin during the session 1905-6, is intended to place the

whole subject before the reader in a connected form. Briefly stated, the following is the order of treatment:—

The book opens with an introductory sketch of definitions and first principles, the proof of Kirchhoff's law, and the definition of a black body. In the second section we have an investigation of Maxwell's formula for the radiation-pressure, Boltzmann's proof of Stefan's law, and Wien's law of distribution of the energy over the different parts of the spectrum, based on the well-known application of a modified form of Doppler's principle. In the last chapter of this section Dr. Planck gives a general discussion of the entropy and temperature of monochromatic radiation. Although the method of treatment is different from that adopted in the introductory part of this review, the conclusions appear to be identical. In particular, it is pointed out that emission without absorption is irreversible, absorption without emission impossible.

In the third part emission and absorption are considered from the point of view of the electromagnetic theory. A resonator is under the influence of periodical or stationary waves. In these circumstances Dr. Planck investigates the oscillations induced in the resonator, and assigns meanings to the entropy and temperature of the resonator which account satisfactorily for reversible phenomena; but the equations of the electromagnetic field being deducible from those of rational dynamics cannot of themselves account for irreversibility, for, corresponding to the solution representing any given process, another solution representing the reverse process can be obtained by changing the sign of the time-differential  $dt$ . Whether the case is stated in this or in some other form, there is no *a priori* reason for asserting that waves cannot converge to a point as readily as they diverge from it. The convergent wave motion simply represents a second solution of the differential equation of propagation, which is commonly omitted merely on the grounds that the corresponding phenomenon does not exist.

The subsequent sections represent an exposition of the valuable work done by Dr. Planck in applying to radiation phenomena the same probability considerations which have led to such fruitful results at the hands of Boltzmann in connection with the kinetic theory of gases. Dr. Planck starts with the assumption that the entropy of a system in a given state depends in some way on the probability of that state, whence it follows that if the system consists of two parts which are independent of each other, and we assume that the entropy of the whole is the sum of the entropies of the parts, the entropy must be a logarithmic function of the probability. A short account of Boltzmann's work for the case of monatomic gases follows, and Dr. Planck then shows how to determine expressions for the entropy of radiation from analogous considerations.

Now Boltzmann's work was not independent of an assumed *a priori* law of probability. He first supposed that for an individual molecule all values of the energy were *a priori* equally probable, and, considering the case of a large assemblage of molecules

the total energy of which was constant, he found that the most probable distribution only agreed with the Boltzmann-Maxwell law in the case in which the molecules were moving in *two-dimensional space*. To obtain the Boltzmann-Maxwell law in other cases it was necessary to start with the assumption that for an individual molecule all values of the *coordinates and momenta* were *a priori* equally probable. If we mistake not, Dr. Planck in § 148 starts with Boltzmann's first assumption. He supposes he has to deal with a large number  $N$  of resonators, that the total energy is divided into a large number  $P$  of equal elements, and that these elements are distributed at random among the resonators perfectly independently of each other. This is, of course, an assumption, but it is shown in § 150 to be equivalent to assuming that all values of the electric and magnetic coordinates of the resonator ( $f$  and  $df/dt$ ) are equally probable. There appears, however, to be an alternative assumption in the case of oscillators distributed in space, namely, that all values of the rectangular components of  $f$  and  $df/dt$  are equally probable, and this might lead to a different result. Would it? and if so, which is right?

In any case, the important fact remains that Dr. Planck obtains results consistent with Stefan's law, notwithstanding that this law cannot possibly represent equipartition of energy at all temperatures between ether and matter. Perhaps the other assumption here suggested would result in equipartition, or the writer of this review has omitted to take account of something in the book. The fifth section is mainly taken up with applications to irreversible processes. In it the consequences of imagining a direct reversal of radiation processes are carefully discussed; the behaviour of an oscillator in a field of radiation is then investigated; the next chapter deals with the conservation of energy and increase of entropy, and, finally, we have a detailed discussion of the particular case of an oscillator exposed to black-body radiation.

It will be thus seen that Dr. Planck's work belongs to a class of investigation which has played an all-important part in building up our knowledge of physical phenomena. It deals with the logical consequences of certain well-defined hypotheses, and as such brings us measurably nearer obtaining a clear and definite idea regarding the irreversible processes associated with radiation. Moreover, the author is careful to define the limitations of his method. As he points out, an unfilled gap still exists in the theory, as he does not fully discuss the tendency to an equilibrium state between oscillations of different periods. The fact that the oscillators are really in motion shows, in connection with Wien's method, that there is a tendency to an equilibrium distribution, and this process may be capable of association with increase of entropy. All readers must express the hope that Dr. Planck may have an opportunity of pursuing this investigation further.

Dr. Planck's book has the great merit of being very readable and intelligible. It is quite easy to see everywhere what the author is driving at; many

points of discussion that a reader would naturally suggest are considered by him exactly where the discussion is wanted, and it will be very easy to criticise the work in the light of any further developments which may arise at the hands of future investigators.

In extending the notion of entropy to radiation, he is working on the lines best calculated to advance our knowledge of thermodynamic phenomena. When a system is in the course of undergoing an irreversible transformation, the entropy at any instant is a definite quantity, provided that at that instant it is possible to conceive a reversible compensating transformation which would bring the system back to its initial state, and also that a definite line can be drawn between the forms of energy that are to be regarded as available and unavailable. In the case of radiation, it is important to push the notion of entropy as far as it will go. Only when this has been done, and the results compared with those of experiment, will it be possible to say whether any limitations exist in the statement that perpetual motion of the second kind is impossible; but the vague and indefinite statements that have hitherto been made regarding the possibility of such motion have mostly been based on the consideration of processes which, when performed cyclically, involve considerable absorption of available energy, and the prospects of obtaining perpetual availability from the ether in ordinary cases of radiation are anything but hopeful.

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#### ELEMENTARY BOTANY.

- (1) *A Text-book of Botany*. By John M. Coulter. Pp. vii+365. (London: S. Appleton, 1906.) Price 5s. net.
- (2) *A First Course in Practical Botany*. By G. F. Scott Elliot. Pp. viii+344. (London: Blackie and Son, 1906.) Price 3s. 6d.
- (3) *First Studies of Plant Life*. By G. F. Atkinson. Edited for use in English Schools by Miss E. M. Wood. Pp. xiv+266. (Boston and London: Ginn and Co., 1905.) Price 2s. 6d.
- (4) *A Text-book of Botany*. Part i. The Anatomy of Flowering Plants. By M. Yates. Pp. v+147. (London: Whittaker and Co., 1906.) Price 2s. 6d. net.

(1) **I**NFLUENCED, it may be, by one's early training, it has always seemed most reasonable to begin an elementary course of botany with morphology, working in so soon as convenient the explanation of form and structure in the light of purpose served, an arrangement that is adopted by Prof. Coulter in his text-book.

The nature and modifications of leaf, stem, and root are first considered, after which the student is directed to the study of unicellular organisms, then to the examination of types of increasing complexity selected from the main taxonomic groups. The summaries of these groups have been carefully prepared, the account of the fungi being specially comprehensive yet brief. Following on the Bryophyta and ferns, a brief sketch of the two generations of the horsetails and lycopods

leads up to the phanerogams. The lessons on flowers and fruits are introduced with the angiosperms, and ecology receives due recognition in the last few chapters.

As the book is liberally supplied with illustrations, chosen with much forethought, it is observable that Prof. Coulter has brought his extensive survey within a remarkably small compass. The merit of the book lies in the judicious selection of essential facts and principles. The numerous references to economic plants constitute a novel feature that is most noticeable in the chapters on the classification of phanerogams. There would be little or no objection to offer if the author made the most of their botanical characters, but these are omitted, and only commercial facts are given, occupying space that would be better filled with botanical information.

Apart from this adverse criticism the book deserves high commendation, and is admirably suited to its purpose for use in secondary schools.

(2) The practical course drawn up by Mr. Scott Elliot presents a somewhat unusual arrangement, inasmuch as the experiments are grouped on a morphological basis; this has, it is true, some advantages, but not sufficient to prefer it to the more usual physiological disposition.

A considerable amount of space is devoted to the flower, as, in addition to one or two chapters, full descriptions of two or more flowers are appended to each day's work. The practice of giving students, when time is available, a few flowers each day is excellent, but it seems unnecessary to provide detailed descriptions of all of them; further, the sequence—if there is a sequence—is not suitable to beginners. The theoretical discussions preceding, but not always relevant to, the practical work are also long, and curtail the space that is allotted to the actual experiments. The experiments, numbering nearly a hundred, are fairly representative, although the fundamental experiments of transpiration, respiration, and osmosis are not so satisfactory or complete as one could have wished. A sufficient amount of anatomical work is included, and a useful bibliography is appended referring the student to accounts of original investigations.

(3) Among the many elementary books dealing with plant life, the studies by Prof. G. F. Atkinson, published about four years ago, have a freshness and vitality of their own. The numerous references to American plants that are not known in this country robbed the book of much of its value for use in schools in the United Kingdom. This defect has been remedied by the introduction of British types, also, we note, of British expressions. Excellent as were the original drawings and photographs, the substitutes prepared by Miss Wood and Dr. J. W. Ellis are quite up to the same standard.

(4) The small volume prepared by Miss Yates is on the lines of the German "Repetitorium," and presents a collection of morphological definitions and terms with illustrations. In the course of thirteen chapters the author collates the different organs of flowering plants, their modifications, and the scientific terms